Education, saving, tourism, imported energy, foreign good, and trade balance of a small open economy

Prof. Wei-Bin ZHANG
Ritsumeikan Asia Pacific University, Japan
E-mail address: wbz1@apu.ac.jp

Abstract
This paper builds an economic growth model with endogenous wealth and human capital accumulation. It analyzes interactions between domestic economic structure, foreign goods, foreign tourism, and imported energy. Education, foreign goods, foreign tourism, and imported energy are integrated into small-open growth models with wealth accumulation in a comprehensive framework. The economic structure is a synthesis of Uzawa’s two-sector growth model and Uzawa-Lucas’ two-sector growth model. The economy is composed of one capital goods sector, services sector, and education sector. The capital goods sector, service sector and households need energies, which foreign economies supply. The capital goods sector, services sector, and households use up the land. The general equilibrium model determines consumption, production, saving, education and resource distribution under given rate of interest and prices of imported energy and goods fixed in international markets. We build the model and simulates its behavior. We conduct comparative dynamic analysis to demonstrate how different exogenous changes change transitory processes and long-run structures of economic growth.

Keywords: human capital; wealth; propensities; price of energy; price of imported goods, propensity to consume foreign goods; economic growth.
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1. Introduction
Interactions between human capital, tourism, energies, and economic growth are basic for understanding mechanisms of economic growth. Although there are theoretical models which deal with interactions between some of these variables, there are only a few models which examine all these variables within a general equilibrium framework. The purpose of this study is to analyze dynamic interdependence between human capital, wealth, tourism, energies, and trade in a small-open economy within general equilibrium framework.

Changes in global prices of energies have significant effects on some economies [1],[2],[3]. Economies such as Japan, Hong Kong, Singapore, and Taiwan, have to import energies in order to sustain their modern development. Increases in prices are pure increases in costs as far as national economies are concerned. As different sectors and households need energies, changes in the price of energy in global market may affect a energy-poor national economy and its economic structures in different ways [4],[5]. Another important factor which is often neglected in theoretical modelling is imported (luxury) goods. The impact of a strong desire for foreign goods is often a concern for national governments as its consumption is perceived to weaken national economy. Economic issues related to foreign goods should be examined within a general equilibrium framework. We develop a general equilibrium model to show the response of economic growth and trade balance to changes in prices of imported energy, cost of capital, and prices of imported goods which are determined in global markets for the small-open economy.
A new feature of contemporary international trade is dramatic increases of global tourism. A few traditional economic theoretical models include tourism as an important determinant of growth. Nevertheless, tourism is obviously a significant determinant of economic growth for some towns, cities, regions or nationals. Tourism goods such as monuments of national heritage, historical sites, beaches, and hot springs, are tradable mainly when tourists come to visit the place. This kind of trade is not analyzed in traditional international trade theory [6]. It is necessary to treat tourism as a separate sector in economic modelling, rather than aggregating tourism with other economic activities [7], [8]. This study introduces tourism an integrated part of economic system. It should be noted that many empirical studies on tourism and economic development are conducted [9], [10], [11], [12]. Researchers have argued for studying dynamic interdependence between tourism and the rest economy within a general equilibrium modeling. This study contributes to the literature on tourism and development within a general equilibrium framework.

This study is also concerned with human capital. Energy-poor advanced economies, like Japan, Hong Kong, Korea and Singapore, can sustain its recent economic development due to their accumulated human capital, through education and other sources. We assume that human capital accumulation mainly comes from education [13], [14], [15], [16], [17], [18], [19]. In addition to the capital goods sector and services sector, the national economy is composed of three sectors. The production side is based on neoclassical growth theory [20], [21]. The education sector follows the Uzawa-Lucas two-sector growth model. International trade are influenced by growth models for small open economies and the literature of growth and tourism. This study is a synthesis of two models recently proposed by Zhang. Zhang [22] studies the impact of imported energy and goods prices on growth and trade balances of a small open economy. Zhang [23] deals with education and tourism in a small open growth economy. We organize the study as follows. Section 2 builds small open economy with endogenous wealth and human capital. Section 3 shows that the system movement is given by two differential equations. Section 4 conducts comparative dynamic analysis to show how the system reacts different exogenous changes. Section 5 concludes the study.

2. The model with imported goods and energy
This study is concerned with economic growth of a small-open economy by synthesizing two models proposed by Zhang [22], [23]. The economic structure is a synthesis of Uzawa’s two-sector growth model and Uzawa-Lucas’ two-sector growth model. It has capital goods sector, services sector, and education sector. The capital goods sector produces a globally free-traded capital goods, which can be either used as capital goods and consumer goods. The service sector produces services, which domestic households and foreign tourists consume. The education sector’s product is consumed only domestic households. All markets are perfectly competitive. The capital goods sector, service sector and households need energies, which foreign economies supply. The capital goods sector, services sector, and households use up the land. Under the assumption of shared ownership, land users pay land rent, which are endogenously determined. We consider the rate of interest and prices of imported energy and goods fixed in international
markets. We built the model and simulated its behavior. To study terms of trade and foreign trade, we introduce imported good, which is not produced by the small-open economy [24]. Imported good is consumed only by domestic households. The economy can import goods and borrow resources from foreign economies, and can export goods and lend resources abroad. We choose the price of the capital good to be unity, with all the prices measured with capital goods. Markets are competitive. Households own all physical wealth and land. Labor, land and capital earn their marginal values. The representative household has incomes from wages, land rent, and interest payments of wealth. Land is only for residential and service use. Tourists are temporary visits without work permission. There is no international emigration or/and immigration. There is a homogeneous fixed population. We introduce

\[ N \] - fixed homogeneous population;
\[ H(t) \] - level of human capital at time \( t \);

subscript index, \( i, s, e \), - industrial, service, and education sector, respectively;
\[ F_j(t) \] - output level of sector \( j \);
\[ K_j(t) \] and \( N_j(t) \) - capital stock and labor force employed by sector \( j \), \( j = i, s, e \);
\[ X_j(t) \] - amount of energy used by sector \( j \);
\[ L \] and \( L_s(t) \) - total land and land used by service sector;
\[ \bar{k}(t) \] and \( \bar{K}(t) \) - capital wealth per household and total capital wealth owned by the population,
\[ \bar{K}(t) = \bar{k}(t)N \];
\[ r^*, P_Z, \text{ and } P_x \] - world interest rate, price of imported goods, price of imported energy, respectively determined in global markets;
\[ p(t) \] and \( p_e(t) \) - price of service and price of education per unit of time;
\[ R(t) \] - land rent;
\[ l(t), c_s(t), \text{ and } c_i(t) \] - lot size, consumption of services, and consumption of capital goods;
\[ c_Z(t), \text{ and } c_s(t) \] - consumption of imported goods and energy consumption;
\[ s(t) \] - saving;
\[ T(t), T(t) \] and \( T_e(t) \) - work time, leisure time, and education time;
\[ T^0 \] - total available time for work, leisure, and education; and
\[ \delta_k \text{ and } \delta_H \] - fixed depreciation rates of physical capital and human capital.

**National labor force**

The national labor force is given by
\[ N(t) = H^m(t)T(t)N, \] (1)
where \( m \) stands for human capital utilization efficiency. The variable \( H^m(t) \) is the level of effective human capital.

**Capital goods sector**

The production function of the capital goods sector is given by

\[
F(t) = A_i k_i^{\alpha_i}(t) N_i^{\beta_i}(t) X_i^{\beta_i}(t), \quad \alpha_i, \beta_i, b_i > 0, \quad \alpha_i + \beta_i + b_i = 1, \quad (2)
\]

where \( A_i, \alpha_i, \beta_i, \) and \( b_i \), are parameters. We have the following the marginal conditions for the sector

\[
r_\delta = \alpha_i A_i k_i^{\alpha_i}(t) x_i^{h_i}(t), \quad w(t) = \beta_i A_i k_i^{\alpha_i}(t) x_i^{h_i}(t), \quad p_x = b_i A_i k_i^{\alpha_i}(t) x_i^{h_i}(t), \quad (3)
\]

where

\[
k_i(t) = \frac{K_i(t)}{N_i(t)}, \quad x_i(t) = \frac{X_i(t)}{N_i(t)}, \quad r_\delta = r^* + \delta_k.
\]

Equations (3) imply

\[
k_i = \left( \frac{b_i^{h_i-1} A_i}{p_x} \right)^{1/\beta_i}, \quad x_i = b k_i, \quad w = \beta_i A_i k_i^{\beta_i} x_i^{h_i}, \quad (4)
\]

where \( b = b_i r_\delta / \alpha_i p_x \).

**Services sector**

There are four inputs, capital, labor force, energy, and land. The production function is specified as

\[
F_s(t) = A_s k_s^{\alpha_s}(t) N_s^{\beta_s}(t) X_s^{\beta_s}(t) L_s^{\beta_s}(t), \quad \alpha_s, \beta_s, b_s, \gamma_s > 0, \quad \alpha_s + \beta_s + b_s + \gamma_s = 1, \quad (5)
\]

where \( A_s, \alpha_s, \beta_s, b_s, \) and \( \gamma_s \) are parameters. We have the following marginal conditions for the service sector

\[
r_\delta = \alpha_s A_s p(t) k_s^{\alpha_s}(t) x_s^{h_s}(t) l_s^{\gamma_s}(t), \quad w = \beta_s A_s p(t) k_s^{\alpha_s}(t) x_s^{h_s}(t) l_s^{\gamma_s}(t),
\]

\[
p_x = b_s A_s p(t) k_s^{\alpha_s}(t) x_s^{h_s}(t) l_s^{\gamma_s}(t), \quad R(t) = \gamma_s A_s p(t) k_s^{\alpha_s}(t) x_s^{h_s}(t) l_s^{\gamma_s-1}(t), \quad (6)
\]
where
\[ k_s(t) = \frac{K_s(t)}{N_s(t)}, \quad x_s(t) = \frac{X_s(t)}{N_s(t)}, \quad l_s(t) = \frac{L_s(t)}{N_s(t)}. \]

Equations (6) imply
\[ k_s = \frac{\alpha_s w}{\beta_s r_s}, \quad x_s = \frac{b_s r_s}{\alpha_s p_x}. \tag{7} \]

**Education sector**

The education sector employs capital and labor as input factors. The production function is specified as
\[ F_e(t) = A_e K_e^{\alpha_e} (t) N_e^{\beta_e} (t), \quad \alpha_e, \beta_e > 0, \quad \alpha_e + \beta_e = 1, \tag{8} \]

where \( A_e, \alpha_e, \) and \( \beta_e \) are parameters. The marginal conditions are
\[ r_s = \alpha_e A_e p_e(t) k_e^{-\beta_e}(t), \quad w = \beta_e A_e p_e(t) k_e^{\alpha_e}(t), \tag{9} \]

where \( k_e(t) = K_e(t) / N_e(t). \) From (7) we have
\[ k_e = \frac{\alpha_e w}{\beta_e r_s}, \quad p_e = \frac{w}{A_e k_e^{\alpha_e}}. \tag{10} \]

**Demand function of foreign tourists**

Let \( y_f(t) \) to represent income in foreign countries. We apply and generalize the iso-elastic tourism demand function by Schubert and Brida [25]
\[ D_T(t) = a(t) \phi \left( \bar{k}(t), H(t), t \right) y_f^\phi(t) p^{-\varepsilon}(t), \tag{11} \]

where \( \phi \) and \( \varepsilon \) are respectively the income and price elasticities of tourism demand. Except price and income, we take account factors, such as national infrastructures, local amenities, and cultural capital [26],[27] which may affect tourists’ demand with variable, \( a(t). \) We assume that tourists consume only services and tourists pay the same price as domestic households, even though this is a strict assumption [28], [29] and [30].

**Optimal decision of domestic households**

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We apply Zhang’s utility function and concept of disposable income [31], [32]. In our approach, households decide education time, leisure time, consumption levels of capital goods, foreign goods, energies, services, and saving. Households are assumed to equally own the national land. Each household thus receives land rent income

\[ \bar{r}(t) = LR(t)/N. \]

We have the current income of the representative household

\[ y(t) = r^* \bar{k}(t) + H^m(t)T(t)w + \bar{r}(t), \quad (12) \]

where \( r^* \bar{k}(t) \) is the interest payment and \( H^m(t)T(t)w \) the wage total payment. The disposable income is the sum of the current income and the value of wealth

\[ \hat{y}(t) = y(t) + \bar{k}(t). \quad (13) \]

The disposable income is distributed between saving and consumption. The budget constraint implies

\[ p_e T_e(t) + R(t)l(t) + p(t)c_z(t) + c_i(t) + p_z c_z(t) + p_x c_x(t) + s(t) = \hat{y}(t), \quad (14) \]

The household is also faced with time constraint

\[ T(t) + \bar{T}(t) + T_e(t) = T_0, \quad (15) \]

Insert (14) in (15)

\[ \bar{p}_e(t)T_e(t) + H^m(t)w\bar{T}(t) + R(t)l(t) + p(t)c_z(t) + c_i(t) + p_z c_z(t) + p_x c_x(t) + s(t) = \bar{y}(t), \quad (16) \]

where

\[ \bar{p}_e(t) \equiv p_e + H^m(t)w, \quad \bar{y}(t) \equiv R^* \bar{k}(t) + H^m(t)T_0 w + \bar{r}(t), \quad R^* \equiv 1 + r^*. \]

We specify utility function \( U(t) \) as follows

\[ U(t) = \theta t^{\nu_0}(t)\bar{T}^{\sigma_0}(t)T_{e,0}^{\eta_0}(t)c_z^{\gamma_0}(t)c_i^{\xi_0}(t)c_x^{\zeta_0}(t)c_{z,0}^{\chi_0}(t)s^{\lambda_0}(t), \]

\[ \nu_0, \sigma_0, \eta_0, \gamma_0, \xi_0, \zeta_0, \chi_0, \lambda_0 > 0, \]

in which \( \nu_0, \sigma_0, \eta_0, \gamma_0, \xi_0, \zeta_0, \chi_0, \lambda_0 \) are propensities to consume lot size, to consume the leisure time, to receive education, to consume services, to consume capital goods, to consume foreign goods, to consume energies, and to hold wealth,
respectively. We maximize \( U(t) \) subject to the budget constraint. The marginal conditions imply

\[
\begin{align*}
l(t) &= \frac{\nu \bar{y}(t)}{R(t)}, \quad \bar{y}(t) = \frac{\sigma \bar{y}(t)}{H^m(t)w}, \quad T_e(t) = \frac{\eta \bar{y}(t)}{p_e(t)}, \quad c_s(t) = \frac{\chi \bar{y}(t)}{p_s(t)}, \quad c_i(t) = \xi \bar{y}(t), \\
c_Z'(t) &= \frac{\zeta \bar{y}(t)}{p_Z}, \quad c_s'(t) = \frac{\chi \bar{y}(t)}{p_s}, \quad s(t) = \lambda \bar{y}(t),
\end{align*}
\]

where

\[
\begin{align*}
\nu &= \rho \nu_0, \quad \sigma = \rho \sigma_0, \quad \eta = \rho \eta_0, \quad \gamma = \rho \gamma_0, \quad \xi = \rho \xi_0, \quad \zeta = \rho \zeta_0, \quad \chi = \rho \chi_0, \\
\lambda &= \rho \lambda_0, \quad \rho = \frac{1}{\nu_0 + \sigma_0 + \eta_0 + \gamma_0 + \xi_0 + \zeta_0 + \chi_0 + \lambda_0}.
\end{align*}
\]

Change in wealth is savings minus dissavings

\[
\dot{k}(t) = s(t) - \bar{k}(t). \quad (17)
\]

**Accumulation of human capital**

We follow Uzawa[13] in modelling human capital accumulation. We apply a generalized Uzawa’s human capital accumulation as follows

\[
\dot{H}(t) = \frac{\nu_e (F_e(t)/\bar{N})^{\pi_e} (H^m(t)T_e(t))^{\psi_e}}{H^{\pi_e}(t)} - \delta_h H(t), \quad (18)
\]

where \( \nu_e, \pi_e, \) and \( \psi_e \) are non-negative parameters. If \( \pi_e \) is positive (negative), we say that learning through education exhibits decreasing (increasing) returns to scale. The equation implies that human capital rises in education service per capita and in the (qualified) total study time, \((H^m(t)T_e(t))^{\psi_e}\).

**Equilibrium in land, input factors, and services markets**

The services sector and households use up the available land

\[
l(t)N + L_s(t) = L. \quad (19)
\]

Full employment of labor force implies
\begin{align*}
N_i(t) + N_s(t) + N_e(t) &= N(t). \tag{20} \\
 \text{Full employment of physical capital implies} \\
K_i(t) + K_s(t) + K_e(t) &= K(t), \tag{21} \\
\text{The equilibrium condition for services is the demand of domestic and foreign tourists} \\
c_s(t)N + D_T(t) &= F_s(t). \tag{22} \\
\textbf{Equilibrium in education market} \\
\text{The equilibrium condition in education markets is} \\
T_e(t)N &= F_e(t). \tag{23} \\
\text{The national wealth is the sum of the households’ wealth} \\
\bar{K}(t) &= \bar{k}(t)N. \\
\text{The trade balance is given by} \\
E(t) &= r^*(\bar{K}(t) - K(t)). \\
\text{The national income is defined as} \\
Y(t) &= F_i(t) + p(t)F_s(t) + p_e(t)F_e(t) + R(t)l(t)N.
\end{align*}

We built the dynamic growth model. The model is a synthesis of the well-known Solow one-sector growth model, Uzawa two-sector model, Uzawa-Lucas education-based growth model, some growth model of small-open economies, and some models on growth and tourism.

\textbf{3.Dynami properties of the small-open economy} \\
The following lemma provides a computational procedure for following movement of the economy. It shows that the dynamics are given by two differential equations. It should be noted that we uniquely determined the variables, $k_i, x_i, k_s, x_s, p_e$, and $w$ as functions of $r^*, p_Z$ and $p_x$ in the previous section.

\textbf{Lemma} \\
Land rent and human capital are determined by two differential equations

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\[ \dot{R}(t) = \Lambda(R(t), H(t)), \]
\[ H(t) = \Omega(R(t), H(t)), \]
\[(24)\]
in which \(\Lambda\) and \(\Omega\) are functions of \(R(t)\) and \(H(t)\) given in the Appendix. All the other variables are determined as functions of \(R(t)\) and \(H(t)\) as follows: \(\tilde{k}(t)\) by (A9) \(\rightarrow p(t)\) by (A8) \(\rightarrow \bar{y}(t)\) by (A9) \(\rightarrow N_e(t)\) by (A10) \(\rightarrow N(t)\) by (A10) \(\rightarrow N_s(t)\) by (A5) \(\rightarrow X_s(t) = x_s(t)N(t)\) \(\rightarrow X_j(t) = x_j(t)N_j(t)\) \(\rightarrow K_j(t) = k_j(t)N_j(t)\) \(\rightarrow N_i(t)\) by (20) \(\rightarrow T_c(t)\), \(\bar{T}(t)\), \(l(t)\), \(c_i(t)\), \(c_j(t)\), \(c_z(t)\), \(c_s(t)\), \(s(t)\) by (12) \(\rightarrow T(t)\) by (15) \(\rightarrow L_s(t)\) by (19) \(\rightarrow F_i(t)\) by (20) \(\rightarrow F_s(t)\) by (5) \(\rightarrow F_c(t)\) by (8) \(\rightarrow K(t)\) by (21).

Following the Lemma, we can determine \(R(t)\) and \(H(t)\) and then determine all the other variables. We simulate the model. We specify tourism demand function as follows

\[ D_T(t) = 0.5 y_f^\phi p^{-\epsilon}(t), \quad y_f = 4, \quad \phi = 1.8, \quad \epsilon = 1.2. \]

According to Syriopoulos[33], income elasticity of tourism demand is larger than unity. Lanza et al.[34] estimate the price elasticity between 1.03 and 1.82 and income elasticities between 1.75 and 7.36[35]. We specify other parameter values as follows

\[ r^* = 0.06, \quad p_Z = 4, \quad p_s = 6, \quad T_0 = 24, \quad \bar{N} = 10, \quad m = 0.6, \quad L = 1, \quad A_i = 1.5, \quad A_s = 1, \quad A_Z = 1, \quad \alpha_s = 0.3, \quad \beta_s = 0.6, \quad \alpha_s = 0.2, \quad \beta_s = 0.5, \quad b_s = 0.1, \quad \alpha_s = 0.4, \quad \lambda_0 = 0.6, \quad v_0 = 0.07, \quad \sigma_0 = 0.15, \quad \xi_0 = 0.15, \quad \gamma_0 = 0.06, \quad \eta_0 = 0.1, \quad \zeta_0 = 0.04, \quad \chi_0 = 0.06, \quad v_e = 0.7, \quad \alpha_e = 0.2, \quad b_e = 0.4, \quad \pi_e = 0.3, \quad \delta_e = 0.05, \quad \delta_i = 0.04. \]

(25)

The population is 10. The rate of interest is specified at 6 per cent. Some empirical studies use the value of the parameter, \(\alpha\), in the Cobb-Douglas production around 0.3. The specified values of the prices, land size and preference parameters will not affect our conclusions of comparative dynamic analysis. We calculate the time-independent variables as follows

\[ w = 0.984, \quad p_e = 0.803. \]

(26)

We choose the initial condition

\[ R(0) = 310, \quad H(0) = 32. \]
The motion of the dynamic system is plotted in Figure 1. The human capital, labor force and national income rise over time. The price of services and land rent fall initially and rise in the long term.

Figure 1 shows how the system approaches an equilibrium point. We calculate the equilibrium values of the variables as follows

\[ w = 0.984, \quad p = 9.28, \quad p_e = 0.8, \quad R = 313.9, \quad H = 33.84, \quad N = 1050.5, \quad Y = 2029.5, \]
\[ X = 64.2, \quad E = -149, \quad K = 4625.3, \quad \bar{K} = 2291.2, \quad N_i = 911.2, \quad N_s = 118.4, \quad N_e = 20.9, \]
\[ K_i = 4077, \quad K_s = 423.7, \quad K_e = 124.6, \quad X_i = 24.9, \quad X_s = 1.09, \quad L_s = 0.149, \quad F_i = 1494.9, \]
\[ F_s = 25.1, \quad F_e = 42.7, \quad \bar{k} = 229.1, \quad c_i = 57.3, \quad c_s = 2.47, \quad c_Z = 3.82, \quad c_e = 3.82, \]
\[ l = 0.085, \quad T = 12.7, \quad \bar{T} = 9.03, \quad T_e = 4.27. \]  

(27)

The eigenvalue at the equilibrium point equals

\[ \{-0.46, \quad -0.04\}. \]

There is a unique stable equilibrium point. This result is important as it guarantees the validity of the following comparative dynamic analysis.

4. **Comparative dynamic analysis**

The previous section demonstrated that we can straightforwardly follow the motion of the economy. We now conduct comparative dynamic analysis. We show how exogenous changes affect transitory processes and long-run equilibrium. As we can follow the motion of the system, it is straightforward to conduct comparative dynamic analysis. We
define a variable, $\Delta x(t)$, to represent the change rate of the variable, $x(t)$, in percentage due to changes in a parameter value.

### 4.1. The price of imported energy rises in global markets

First, we examine what happens to the economy if the global price of energies is increased as: $p_x : 6 \Rightarrow 6.5$. The effects on the wage rate and price of education which are independent of time are reduced as follows:

$$\Delta w = -1.32, \quad \Delta p_e = -0.8.$$  

An increase in imported energy price lowers wage rate and price of education. The impact on the time-dependent variables are plotted in Figure 2. The economy imports less energies. The households and the sectors which use energies are reduce amount of energies. The net impact on trade is more trade surpluses. Human capital is slightly affected. The labor force falls initially and changes little in the long term. As also explained in Hamilton [36], [37] energy price increases cause economic recessions. The national output falls. The economy uses less capital and owns less wealth in the long term. The output of the education sector changes slightly. The other two sectors’ output levels fall. The land distribution and time distribution are slightly affected in the long term. The price of services and land rent fall. The household owns less wealth, consumes less capital goods, services and imported goods. As the services price falls, tourism income is increased.

![Figure 2](image-url)

**Fig. 2.** The price of imported energy rises

### 4.2. The propensity to consume imported goods is increased

We now examine the case that the propensity to consume imported goods is enhanced follows: $\sigma_0 : 0.04 \Rightarrow 0.05$. The change has no impact on the wage rate and price of
education. Figure 3 shows the impact on the time-dependent variables. The household consumes more imported goods and less energies, services and capital goods. The household also has less wealth. The land distribution is slightly affected. The household spends more time on work and less hours on leisure and education. The human capital falls in association with falling hours in receiving education. The economy employs more capital and has less wealth. The national labor force and national income are enhanced. The amount of energies imported by the economy falls. The capital goods sector uses more energies, while the services sector uses less energies. The capital goods sector is expanded, while the other two sectors are shrunk. The price of services and land rent fall.

Fig. 3. The propensity to consume imported goods is increased

4.3. The rate of interest rises in global markets
We now show how the economy reacts to the following change in the rate of interest in global markets: \( r^* = 0.06 \Rightarrow 0.065 \). The wage rate falls and price of education rises as follows:

\[ \Delta w = -2.2, \quad \Delta p_e = 0.45. \]

Figure 4 shows the impact on the time-dependent variables. The labor force and national output are reduced as capital costs are increased in global markets. As shown in the figure, the economy as a whole suffers from the rising cost of capital.
4.4. The propensity to save is increased

The propensity to save is shifted as follows: $\lambda_0 = 0.6 \Rightarrow 0.65$. We have $\Delta w = \Delta p_e = 0$. Figure 5 plots the impact on the time-dependent variables. As the propensity to save is enhanced, the economy has more wealth. The household spends more hours in education and leisure, but less hours on work. Although the human capital is improved, the national labor force is reduced because of falling in work hours. The household has more wealth, consumes more services, imported goods and energies. The economy employs less capital. The land rent and price of services are increased. The national output falls. Although the services and education sectors are expanded, the capital goods sector shrinks. The trade balance is improved.
4.5. The household enhances the propensity to receive education

The propensity to receive education is shifted as follows: \( \eta_0 = 0.1 \Rightarrow 0.11 \). We have \( \Delta w = \Delta p_e = 0 \). Figure 6 plots the impact on the time-dependent variables. The household spends more hours on education, but less hours on leisure and work. The human capital rises. The national labor force and national output fall initially and rise in the long term. The economy initially uses less energies and more in the long term. The two sectors and household consume less energies and more in the long term. The three sectors are expanded in the long term. The price of services and land rent fall initially and rise in the long term. Similarly, the household consumer less services, capital goods and imported goods, but more in the long term. We see that although a rise in the propensity to receive education does not benefit the household in the short term but does benefit in the long term.

![Figure 6. The household enhances the propensity to receive education](image)

4.6. The total factor productivity of the capital goods sector is enhanced

The propensity to receive education shifts as follows: \( A_i = 1.5 \Rightarrow 1.6 \). The wage rate rises and the price of education rises as follows

\[ \Delta w = 11.4, \quad \Delta p_e = 6.67. \]

Figure 7 plots the impact on the time-dependent variables. The time and land distributions are slightly affected. From the figure we see that the household benefits in the long term.
5. Conclusions

This paper built an economic growth model with endogenous wealth and human capital accumulation. It built a dynamic economic model with complicated interactions between domestic economic structure, foreign goods, foreign tourism, and imported energy. Education, foreign goods, foreign tourism, and imported energy are integrated into small-open growth models with wealth accumulation in a comprehensive framework. The economic structure is a synthesis of Uzawa’s two-sector growth model and Uzawa-Lucas’ two-sector growth model. It has capital goods sector, service sector, and education sector.

The capital goods sector produces a globally free-traded capital goods, which can be either used as capital goods and consumer goods. The service sector produces services, which domestic households and foreign tourists consume. The education sector’s product is consumed only domestic households. The capital goods sector, service sector and households need energies, which foreign economies supply. The capital goods sector, services sector, and households use up the land. Under the assumption of shared ownership, land users pay land rent, which are endogenously determined. We build a general equilibrium model to determine consumption, production, saving, education and resource distribution under given global economic conditions. Global markets are free in capital goods, energies and foreign goods. Like many models in the literature of small-open economic growth, we consider the rate of interest and prices of imported energy and goods fixed in international markets. We built the model and simulated its behavior. As the model is developed in a general equilibrium framework, we can analyze impact of any change in an exogenous condition on the economic system over transitory processes and the long-run equilibrium point. We conducted comparative dynamic analysis with regard to different exogenous changes. The general equilibrium model developed in this study may be generalized in different directions. For instance, rather than a small-open economy, we may discuss dynamics of multiple economies. We may also introduce different possible government interventions in trade. The main determinants of growth...
are education, wealth and trades. There are other important problems, such as R&D and heterogeneous households. It will provide more insights into dynamic processes of economic development if we take account of these determinants.

Appendix: Proving the Lemma

We solved $k_i, w, k_e, s, p_e$ and $x_s$ as functions of $r^*, p_z$ and $p_s$. In the Appendix we omit time variable. From $K_j = k_j N_j$ and (21), we get

A.1. $k_i N_i + k_s N_s + k_e N_e = K$.

From (6) and $l_s = L_s / N_s$, we have

$$R = \frac{w_s N_s}{L_s},$$

A.2.

where $w_s = w \gamma_s / \beta_s$. Insert (A2) in (15)

$$l - \bar{N} + \frac{w_s N_s}{R} = L.$$

A.3.

The definition of $\bar{y}$ implies

$$\bar{y} = R^* \bar{k} + H^m T_0 w + \frac{RL}{\bar{N}}.$$

A.4.

By (A4) and $l = \nu \bar{y} / R$ in (16), we get

$$l = \frac{R^* \nu \bar{k} + \nu H^m T_0 w}{R} + \frac{\nu L}{\bar{N}}.$$

A.5.

Insert the above equation in (A3)

$$R^* \nu \bar{k} + \nu H^m T_0 w + \frac{w_s N_s}{\bar{N}} = (1 - \nu) \frac{RL}{\bar{N}}.$$

A.5.

By $r_\delta = \alpha_s p F_s / K_s$ and (22), we have
\[ c_s \bar{N} + D_T = \frac{r_\delta K_s}{\alpha_s p}. \]

(A.6.)

Substituting \( c_s = \gamma \bar{y} / p \) into (A6) yields

\[ \gamma \bar{y} \bar{N} + p D_T = \frac{r_\delta K_s}{\alpha_s}. \]

Insert (A4) in the above equation

\[ R^s \bar{k} + H^m T_0 w + \frac{R L}{\bar{N}} + \frac{p D_T}{\gamma \bar{N}} = \frac{r_\delta K_s}{\gamma \bar{N} \alpha_s}. \]

(A.7.)

By (6) and \( I_s = w_s / R \), we have

\[ p(R) = \frac{R^s}{\gamma_s A_s w_s^{\gamma_s - 1} k_s^{\alpha_s} x_s^{b_s}}. \]

(A.8.)

From (A7) and (A5), we solve

\[ \bar{k} = \varphi(R, H) \equiv \frac{1}{(\nu + w_s \bar{y}) R^s \bar{N}} \left[ (1 - \nu) R L - \bar{y} w_s R L - \nu H^m T_0 \bar{N} w - H^m w_s \bar{y} T_0 w - \bar{y} w_s p D_T \right], \]

(A.9.)

where we use \( K_s = k_s N_s \) and \( \bar{y} = \gamma \alpha_s / r_\delta k_s \).

From (23), (8) and (6), we have

\[ N_e = R^s \bar{p} \bar{k} + \bar{p}_0, \]

(A.10.)

where we apply (A4) and

\[ \bar{p} = \frac{\eta \bar{N}}{A_e k_e^{\alpha_e} \bar{p}_e}, \quad \bar{p}_0 = H^m T_0 \bar{p} w + \frac{\bar{p} R L}{\bar{N}}. \]

From (15) and (1), we have
A.11. \( N = H^m T_0 \bar{N} - \bar{\eta} \bar{y}, \)

where we also use (16) and

\[
\bar{\eta} = \left( \frac{\eta}{\bar{P}_e} + \frac{\sigma}{H^m w} \right) H^m \bar{N}.
\]

A.12.

Insert (A4) in (A12)

\[ A.13 \ N = n - \bar{\eta} R^* \bar{k}, \]

where

\[
n = H^m T_0 \bar{N} - \bar{\eta} H^m T_0 w - \frac{R\bar{\eta}L}{\bar{N}}.
\]

We can thus determine variables as functions of \( R \) and \( H \): \( k \) by (A9) \( \rightarrow P \) by (A8) \( \rightarrow \bar{y} \) by (A9) \( \rightarrow N_e \) by (A10) \( \rightarrow N \) by (A10) \( \rightarrow N_s \) by (A5) \( \rightarrow X_s = x_s N_s \) \( \rightarrow K_j = k_j N_j \) \( \rightarrow N_i \) \( \rightarrow l, T_e, T, c_i, c_s, c_z, s \) by (20) \( \rightarrow I \) \( \rightarrow L \) by (15) \( \rightarrow F \) by (19) \( \rightarrow F \) by (12) \( \rightarrow T \) by (15) \( \rightarrow L \) by (19) \( \rightarrow F \) by (20) \( \rightarrow F \) by (12) \( \rightarrow T \) by (15) \( \rightarrow L \) by (19) \( \rightarrow F \) by (20) \( \rightarrow F \) by (12) \( \rightarrow T \) by (15) \( \rightarrow L \) by (19) \( \rightarrow F \) by (21).

From this procedure, (17) and (18), we have

\[ A.14 \ \frac{\dot{k}}{\lambda} = \Lambda_0(R, H) \equiv \lambda \bar{y} - \bar{k}, \]

\[
\dot{H} = \Omega(R, H) \equiv \frac{v_e (F_e / \bar{N})^{\psi_e} (H^m T_e)^{\psi_e}}{H^{\psi_e}} - \delta_h H.
\]

A.15.

Take derivatives of (A9) in time

\[
\frac{\dot{k}}{\dot{R}} = \frac{\dot{H}}{H} + \Omega \frac{\dot{H}}{H}.
\]

A.16.

From (A14) and (A16), we have
\[
\dot{R} = \Lambda(R, H) \equiv \left( \Lambda_0 - \Omega \frac{d\varphi}{dH} \right) \left( \frac{d\varphi}{dR} \right)^{-1}.
\]

A.17.

We proved the lemma.

References