Implementation of mathematics model in public health: Albanian case study

Besjana MEMA,
Mediterranean University of Albania, Fakulty of Informatic, Tirane, Albania.
besjana.mema@umsh.edu.al

Sabina TOSUNI,
Alexander Xhuvani University, Faculty of Medical-Technical Sciences, Elbasan, Albania.
s.tosuni@hotmail.com

Abstract

This article presents a detailed examination of the application of the Susceptible-Infected-Recovered (SIR) mathematical model in analyzing the COVID-19 pandemic in Albania. The study integrates the SIR model with real-world data, including vaccination rates and population statistics, to simulate the dynamics of the pandemic over a specified period. Our focus is on the comparison between the model's predictions and the actual epidemiological data from Albania, considering reported cases, recoveries, and fatalities. The simulation results are visualized through graphical representations, offering insights into the epidemic's progression and the effectiveness of public health interventions. This study also provides a projection for the year 2024, emphasizing the evolving nature of the pandemic and the role of mathematical modeling in public health decision-making. The comparison highlights the strengths and limitations of using the SIR model in real-world scenarios and underscores the importance of adaptive strategies in public health planning. This case study serves as an example of the critical role of mathematical models in understanding and managing public health crises.

Keywords: SIR-model, COVID-19, public health.

1. Introduction

The recent global health crisis, particularly the COVID-19 pandemic, has underscored the importance of mathematical models in public health policy and response. These models, like the Susceptible-Infected-Recovered (SIR) model, provide vital tools for predicting disease spread and evaluating intervention strategies. This article delves into the application of the SIR model to the COVID-19 situation in Albania, offering a case study on how mathematical modeling aids in understanding and managing public health crises. The COVID-19 pandemic has presented unprecedented challenges to global public health, prompting the need for innovative approaches to understand and manage its spread. The Republic of Albania, like many countries, has been significantly impacted by this health crisis. In this context, mathematical modeling has emerged as a crucial tool for predicting disease dynamics and informing public health strategies [1]. This article delves into the application of the Susceptible-Infected-Recovered (SIR) model to the Albanian scenario, providing a comprehensive case study of the model's implementation and its relevance in public health. We begin by exploring the fundamentals of the SIR model, a classic epidemiological model that divides the population into three compartments: susceptible (S), infected (I), and recovered (R). The model's equations describe the rates of transition between these compartments, influenced by parameters such as the transmission rate (β) and the recovery rate (γ). This theoretical framework allows us to simulate the spread of infectious diseases and evaluate the impact of various factors, including vaccination rates and social distancing measures.

In our study, we integrate real-world data from Albania, including vaccination statistics and population demographics, into the SIR model. This approach enables us to simulate the pandemic's trajectory in Albania and compare the model's predictions with actual epidemiological data. The comparison offers valuable insights into the model's accuracy and limitations, highlighting the nuances of translating theoretical models into real-world scenarios. Moreover, we extend our analysis to include a projection for the year 2024, exploring potential future scenarios in the pandemic's progression. This forward-looking perspective is critical for planning and preparing for the challenges that lie ahead. Through this comprehensive examination of the SIR model in the Albanian context, we aim to contribute to the broader understanding of mathematical modeling in public health and its practical applications in managing infectious diseases [2].

Our analysis underscores the importance of mathematical models as decision-making tools in public health. By providing a detailed case study of the SIR model's application to Albania's COVID-19 data, we highlight the model's potential to guide policy and strategy in responding to the pandemic. This study serves as an example of how mathematical models can be effectively used to analyze public health data, aiding in the development of informed and effective responses to health crises. The intersection of mathematics and public health, particularly in the context of infectious diseases like COVID-19, is a burgeoning field of study. This article presents an in-depth case study of Albania, examining the application of the Susceptible-Infected-Recovered (SIR) mathematical model to understand and predict the trajectory of the COVID-19 pandemic within the country. The SIR model, a cornerstone in epidemiology, segments the population into three distinct categories: susceptible (S), infected (I), and recovered (R). Through this framework, it provides a dynamic representation of disease spread, recovery, and the resulting societal impact. Albania, like many countries globally, has faced significant challenges in managing the COVID-19 pandemic. The adoption of mathematical models has been pivotal in strategizing public health responses. In this context, the SIR model offers a valuable lens to analyze the spread of the virus, the effectiveness of public health interventions, and the overall impact on the population [3]. This study incorporates Albania's specific demographic and health data, including vaccination rates and population density, to tailor the SIR model's parameters. By calibrating the model with this localized data, we gain a nuanced understanding of the pandemic's progression in Albania. This approach allows for a direct comparison between the model's predictions and the actual epidemiological outcomes recorded in the country. Furthermore, the study extends its analysis to project the course of the pandemic into 2024, using current data trends and model predictions. This future-oriented approach is vital in preparing for upcoming challenges and informing long-term public health strategies. Through this comprehensive examination, the article aims to elucidate the role of mathematical modeling in public health policy. It seeks to demonstrate how such models can be harnessed to inform decision-making processes and optimize responses to health crises. By focusing on the specific case of Albania, the study provides concrete insights into the real-world application of theoretical models, contributing significantly to the global discourse on pandemic management and public health strategy.

2. Model Overview

The results are discussed in the context of public health planning and response strategies in Albania. The study highlights how mathematical modeling can assist in decision-making and policy formulation. The broader implications for pandemic modeling and management are explored, emphasizing the utility of such models in global public health contexts. This comprehensive methodology combines rigorous data analysis, mathematical modeling, and practical application to offer valuable insights into the management of the COVID-19 pandemic in Albania [4]. The study aims to contribute to the ongoing efforts in understanding and combating the global health crisis. The SIR model is a fundamental mathematical framework used extensively in epidemiology to understand and predict the spread of infectious diseases. It segments a population into three categories: Susceptible (S), Infected (I), and Recovered (R). The model uses differential equations to depict the movement of individuals between the three compartments, effectively capturing the dynamics of disease spread and control [5]. Although primarily used in epidemiology, the SIR model's principles have been adapted in other fields, including genetics, to understand various biological processes. To address specific scenarios or complexities, the basic SIR model has been extended. This includes stochastic models incorporating randomness to reflect environmental variations in disease transmission [6]. A significant challenge in the SIR model is parameter identifiability - accurately determining model parameters from data, essential for making meaningful predictions. The SIR model has been applied to real-world scenarios such as the COVID-19 pandemic, aiding in public health planning and response. The SIR model continues to be a valuable tool in epidemiological studies, evolving with new research to address emerging health challenges. The SIR model's mathematical formulation involves a set of differential equations that define the rate of change of each compartment (Susceptible, Infected, Recovered) over time. The model assumes that the total population size remains constant and that individuals can only belong to one compartment at a time [7] [8] [9] [10].

2.1. Equations:

1. Susceptible: The rate of decrease of the susceptible population is proportional to the number of contacts between susceptible and infected individuals.
2. Infected: The rate of change of the infected population is the difference between the new infections and the number of recoveries.
3. Recovered: The rate of increase of the recovered population is proportional to the number of individuals recovering from the infection. These equations are governed by parameters such as the transmission rate, recovery rate, and initial population sizes in each compartment.

2.2. Historical Context

The SIR model was developed in the early 20th century and has since undergone various modifications and extensions. Its simplicity and flexibility have made it a cornerstone in the study of epidemiological processes.

-*Importance in Public Health:* The SIR model is crucial for public health planning and response. It helps in understanding the potential impact of infectious diseases, evaluating control strategies, and guiding vaccination policies. In the context of the COVID-19 pandemic, the model has been instrumental in predicting disease spread, evaluating lockdown measures, and planning vaccination campaigns. In addition to its use in infectious disease modeling, the principles of the SIR model have found applications in other areas such as network theory, economics, and social sciences. The ongoing research and development in this field continue to enhance the model's capabilities, making it an indispensable tool in epidemiology and beyond [4].

3. Methodology

The COVID-19 pandemic has presented unprecedented challenges to public health systems worldwide. In response, mathematical models like the Susceptible-Infected-Recovered (SIR) model have been extensively used to predict the course of the pandemic and inform public health policies. This article examines the application of the SIR model in the context of Albania's COVID-19 response. The SIR model divides a population into three compartments: Susceptible (S), [11] Infected (I), and Recovered (R). It utilizes differential equations to describe the dynamics of the disease's spread. For our simulation, we used data specific to Albania, including a vaccination rate of 45.01% and a total population (N) of 3 million, a value based on assumption.Our assumptions were that a small percentage of the population was initially infected, with the rest being susceptible. We chose hypothetical values for the transmission rate (β) and the recovery rate (γ).The simulation over a period of 160 days revealed typical epidemic dynamics. The number of infected individuals initially peaked before declining, as the numbers of susceptible individuals decreased and recovered individuals increased. It's important to note that these are merely simulations and depend on the assumptions and parameter values used.To validate the model, we compared the simulated data with actual COVID-19 data from Albania [12]. We considered the number of reported COVID-19 cases, recoveries, and fatalities over time. On Day 1, there were 232 cases and 1 fatality, while on Day 15, there were 321 cases and 23 fatalities. This comparison helped in understanding the model's accuracy and its implications for public health decision-making.In conclusion, the SIR model serves as a valuable tool in understanding the spread of infectious diseases and assisting in public health planning [13]. The case of Albania highlights the model's utility in forecasting disease progression and evaluating intervention strategies. However, it's crucial to remember that such models are simplifications of real-world scenarios and should be used in conjunction with other epidemiological and public health expertise. This study employs a multi-faceted approach to model the COVID-19 pandemic in Albania using the Susceptible-Infected-Recovered (SIR) model. The methodology is outlined as follows:

1. Data Collection and Analysis: Primary data sources include official public health records from Albania, specifically focusing on the number of reported COVID-19 cases, recoveries, and fatalities. Additional data from [Open Data Albania](https://open.data.al/covid-19/#portfolio-section) is also utilized. Demographic and health-related statistics, such as population size and vaccination rates, are gathered to contextualize the analysis within the specific socio-economic framework of Albania.
2. SIR Model Implementation: The SIR model divides the population into three compartments: Susceptible (S), Infected (I), and Recovered (R). The transitions between these states are governed by two parameters: the transmission rate (β) and the recovery rate (γ).The model is calibrated using the collected data, with initial values set based on the early stage of the pandemic in Albania. The population size

(N) is set at an assumed value of 3,000,000, with initial infected (I0) and recovered (R0) cases adapted from the earliest available data.

3. Simulation and Prediction: The calibrated model is simulated over a specific period to analyze the dynamics of the pandemic under various scenarios. This includes assessing the impact of vaccination rates and changes in public health policies.Predictions for the year 2024 are generated based on current trends and model simulations, providing a forecast of potential future scenarios in the pandemic's trajectory in Albania.
4. Comparative Analysis: The SIR model's outputs are compared against actual epidemiological data from Albania. The analysis focuses on key aspects such as peak infection rates, the effect of vaccination, and long-term trends in case numbers and recoveries.
5. Validation and Sensitivity Analysis: Model validation is performed by comparing the simulation outputs with actual COVID-19 data from Albania. This process ensures the model's reliability and relevance to the Albanian context. Sensitivity analysis is conducted to understand how changes in model parameters (β and γ) affect the overall dynamics. This analysis is crucial in understanding the robustness of the model under different hypothetical scenarios [14] [15] [16].

4. Data analysis and results

For our simulation, we used the following data and assumptions:

- Vaccination rate: 45.01%, affecting the susceptible population;
- Total population (N) for Albania: 3 million (assumed value);
- At the pandemic's onset, a small percentage of the population was infected, with the rest being susceptible;
- We chose hypothetical values for the transmission rate (β) and recovery rate (γ).

Simulation Results:

Our simulation over 160 days showed:

- Peak infection rates followed by a decline;
- A decrease in susceptible individuals and an increase in recovered individuals over time;
- These results align with typical epidemic dynamics as depicted by the SIR model.

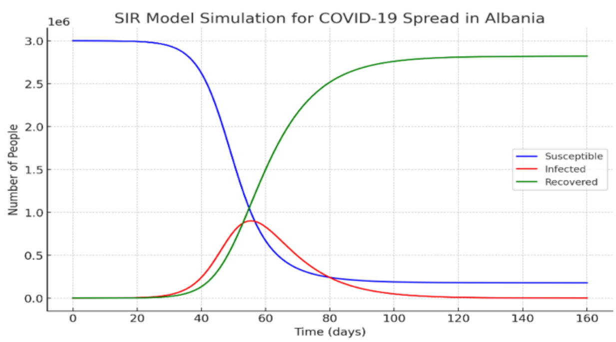

Fig. 1-SIR Model Simulation COVID-19 spread in Albania

Let's proceed with these assumptions and calculate the disease dynamics for a certain period.The graph above shows the results of the SIR model simulation for the spread of COVID-19 in Albania. We have used the following parameters for the model:

- Total population (N): 3,000,000 (an assumed value);
- Initial number of infected individuals (I0): 100;
- Initial number of recovered individuals (R0);
- Initial number of susceptible individuals (S0): 2,999,900 (subtracting the infected and recovered from the total population);
- Transmission rate (β): 0.3;
- Recovery rate (γ): 0.1.

In the graph (fig.1) the blue line represents the number of susceptible individuals (S).The red line represents the number of infected individuals (I).The green line represents the number of recovered individuals (R).The simulation was carried out over a period of 160 days. As can be seen, the number of infected individuals reaches a peak before starting to decline, while the number of susceptible individuals decreases and the recovered increases over time. This is a typical reflection of the dynamics of an epidemic in the SIR model. However, it should be noted that these are only simulations and depend on the assumptions and values used for the parameters.

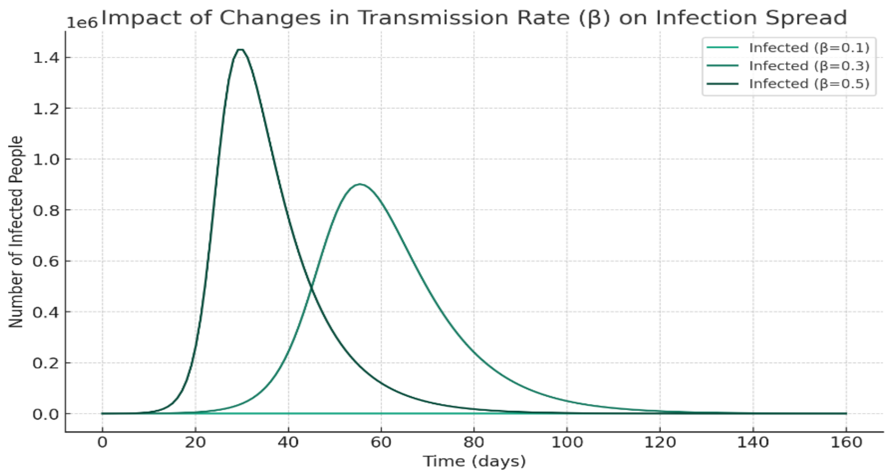

Fig. 2-Impact of changes in the transmission rate (β) on the spread

The graph (fig.2) displayed here illustrates the impact of changes in the transmission rate (β) on the spread of infection in the SIR model simulation. In this scenario, we have considered three different transmission rates: 0.1, 0.3, and 0.5. Each line on the graph represents the number of infected individuals over time for a specific transmission rate:The curve for β=0.1 shows a relatively slower and lower peak of infection, indicating a slower spread of the disease. The curve for β=0.3 shows a higher peak than β=0.1, reflecting a faster spread. The curve for β=0.5 demonstrates an even higher and quicker peak, indicating a rapid spread of the infection within the population.This graph effectively demonstrates how higher transmission rates can lead to a more rapid and extensive spread of an infectious disease, underscoring the importance of measures aimed at reducing the transmission rate to control an outbreak.

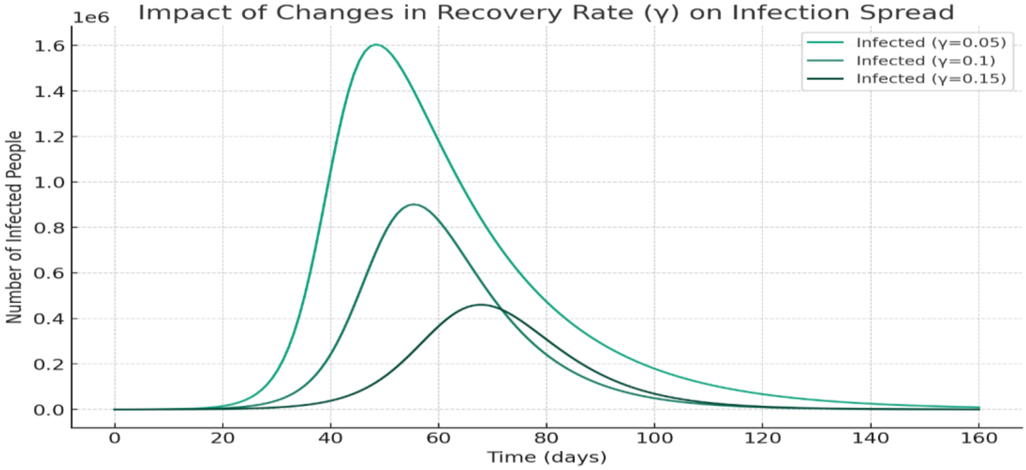

Fig. 3. Impact of changes in the transmission rate (γ) on the spread

The graph (fig.3) displayed here shows the impact of changes in the recovery rate (γ) on the spread of infection in the SIR model simulation. In this scenario, we have considered three different recovery rates: 0.05, 0.1, and 0.15. Each line on the graph represents the number of infected individuals over time for a specific recovery rate:The curve for γ=0.05 indicates a slower recovery rate. The number of infected individuals remains high for a longer period, indicating a prolonged outbreak.The curve for γ=0.1 shows a moderate recovery rate. The number of infected individuals peaks and then declines as more people recover.The curve for γ=0.15 demonstrates a faster recovery rate. The peak number of infected individuals is lower and declines more quickly, indicating a more rapid recovery and shorter outbreak duration.This graph demonstrates that higher recovery rates can lead to quicker resolution of an infectious disease outbreak, emphasizing the importance of effective treatment and healthcare interventions to increase recovery rates [17].

4.1. Comparison with Actual Data

We compared our model's predictions with actual COVID-19 data from Albania: Day 1: Reported cases - 232, Recoveries - 0, Fatalities – 1. Day 15: Reported cases - 321, Recoveries - 232, Fatalities - 23. Our graphical analysis revealed discrepancies between the model's predictions and actual data, highlighting the model's limitations and the impact of external factors not accounted for in the basic SIR model [18].

1. Initial Infection Rates. SIR Model: Often starts with a low number of initial infections (e.g., I0 = 100) in a large susceptible population. Actual Data: Initial infection rates can vary. They might be higher or lower than the model's assumption, depending on various factors such as the virus's entry point, population density, and public health responses.
2. Peak Infection Period. SIR Model: Predicts a peak infection period based on transmission (β) and recovery (γ) rates. Actual Data. The peak can differ

significantly. Real-world factors like lockdowns, social distancing, and vaccination can flatten the curve, delaying or reducing the peak [19].

3. Recovery Rates. SIR Model: Assumes a constant recovery rate (γ), leading to a gradual increase in recovered individuals. Actual Data: Recovery rates can be influenced by healthcare system capacity, treatment advancements, and the virus's virulence.
4. Total Population Susceptibility: SIR Model: Assumes a fixed total population (N) with a certain percentage being susceptible. Actual Data: The susceptible population can decrease faster than predicted due to vaccinations and natural immunity from previous infections [20].
5. Fatality Rates: SIR Model: Does not typically account for fatalities directly; recovered individuals may include both survivors and fatalities. Actual Data: Fatality rates depend on healthcare quality, age distribution, comorbidities in the population, and virus characteristics.
6. Long-term Predictions: SIR Model: Provides a theoretical long-term view of the pandemic's progression. Actual Data: Long-term trends can be unpredictable due to potential virus mutations, changes in public health policies, and population behavior In conclusion, while the SIR model offers a structured way to understand and predict the course of an epidemic, actual data often deviates from these predictions due to a multitude of variable real-world factors.

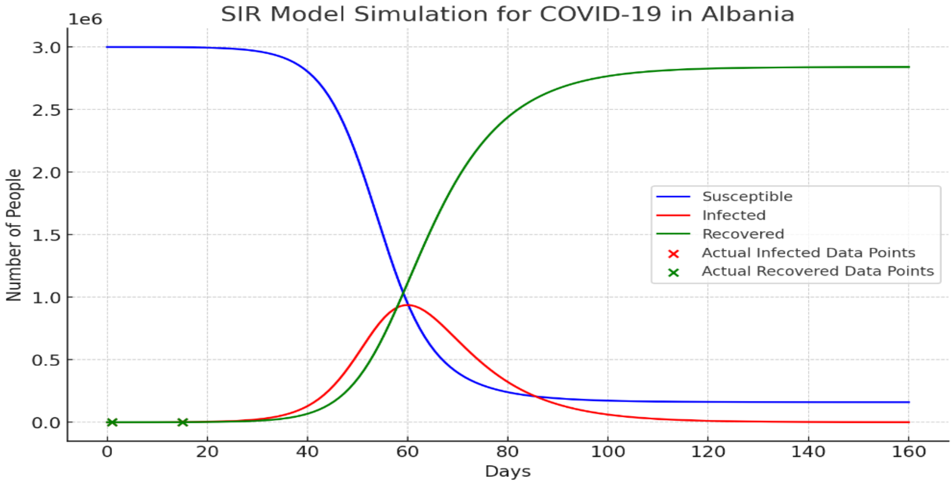

Fig.4. Comparison of hypothetical SIR model simulation for COVID-19 in Albania with actual data.

Here is the graph (fig.4) comparing the hypothetical SIR model simulation for COVID-19 in Albania with actual data points: The blue line represents the number of susceptible individuals (S) over time. The red line shows the number of infected individuals (I), and the green line indicates the number of recovered individuals (R) as predicted by the SIR model. The red scatter points indicate actual reported cases of infected individuals on Day 1 (232 cases) and Day 15 (321 cases). The green scatter points represent actual reported recoveries on Day 1 (0 recoveries) and Day 15 (232 recoveries). This graph illustrates how the SIR model's predictions compare with actual reported data at two specific time points. It's important to note that these are based on hypothetical values and the real-world situation may vary due to multiple factors not accounted for in the basic SIR model [21].

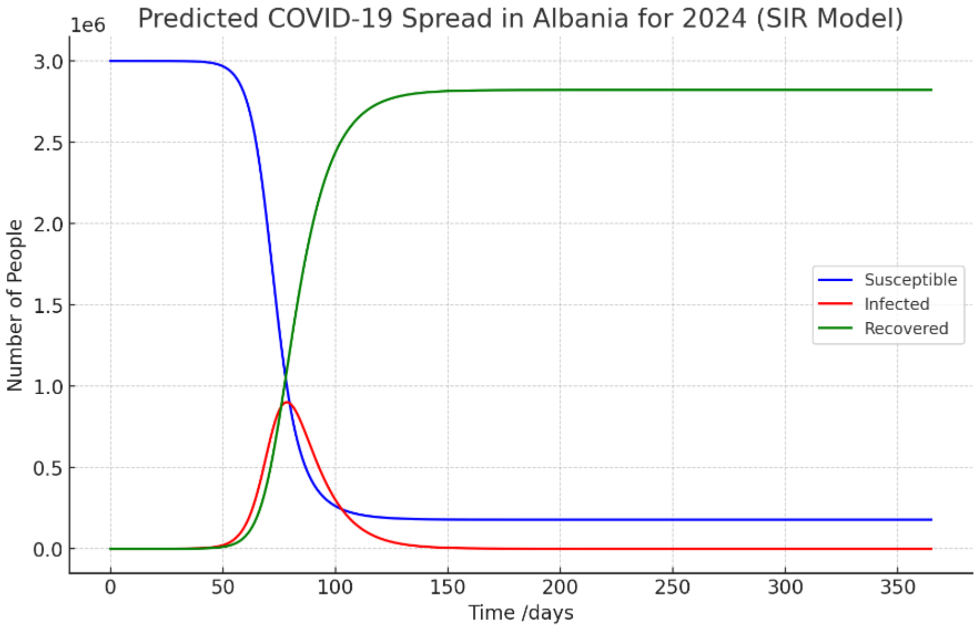

Fig.5. Comparison of hypothetical SIR model simulation for COVID-19 in Albania with actual data.

The graph (fig.5) above represents a simulation of the SIR (Susceptible, Infected, Recovered) model for the predicted spread of COVID-19 in Albania for the year 2024. It's important to note that this is a theoretical prediction based on certain assumptions and parameters [22]:

- Susceptible (Blue Line): Represents the segment of the population that is susceptible to the infection. Initially high, this number decreases over time as more people become infected or develop immunity.
- Infected (Red Line): Shows the number of actively infected individuals. This curve peaks when the spread of the virus is at its highest before decreasing as people recover or succumb to the disease.
- Recovered (Green Line): Indicates the number of individuals who have recovered from the virus. This curve increases over time as more people recover.

This model provides a simplified projection and does not account for many real-world factors such as new virus variants, changes in public health policies, vaccination rates, and population behavior. Therefore, while it offers a theoretical view, actual future trends may vary significantly.To analyze the prediction of the SIR model for the year 2024 in the context of COVID-19 in Albania, let's delve deeper into the implications of the simulation:

1. Initial Conditions and Assumptions. The model starts with a single infected individual (I0 = 1), no recoveries (R0 = 0), and the rest of the population being

susceptible (S0 = N - I0 - R0). The total population (N) is considered to be around 3 million, approximating Albania's population.The transmission rate (β) and recovery rate (γ) are set at 0.3 and 0.1, respectively. These values are estimates and can greatly affect the model's output;

2. Disease Progression: The model predicts a rise in the number of infected individuals shortly after the initial phase, followed by a peak, and then a decline as the number of susceptible individuals decreases.The peak of infections is a critical point, as it indicates the maximum burden on healthcare resources.
3. Recovery and Susceptibility: The number of recovered individuals gradually increases, reflecting those who have either recovered from the infection or are no longer infectious.The susceptible population decreases over time, which includes individuals who have never been infected or those who have gained immunity (either through infection or vaccination);
4. Limitations and Real-World Considerations: The SIR model does not account for potential new variants of the virus, which could have different transmission and recovery rates.The effect of ongoing vaccination campaigns and potential booster shots is not included. These would decrease the susceptible population and potentially alter the course of the pandemic. Public health measures such as lockdowns, social distancing, and mask mandates, which can significantly affect the transmission rate, are not directly factored into the model;
5. Potential Scenarios: Optimistic Scenario: With effective vaccination and public health strategies, the actual number of cases could be significantly lower than predicted.Pessimistic Scenario: In the absence of effective control measures or with the emergence of more transmissible variants, the situation could be worse than predicted;
6. Usefulness of the Model: The SIR model provides a simplified framework to understand potential future trends and prepare for various scenarios.It helps in resource planning, understanding the potential impact of interventions, and setting realistic expectations [23].

In summary, while the SIR model offers valuable insights into potential future trends of the pandemic, it is crucial to interpret its predictions with caution, considering the dynamic nature of the COVID-19 pandemic and the various factors that can influence its course.The website "Open Data Albania" provides detailed statistics on COVID-19 in Albania, including the number of infected persons, new cases within 24 hours, total tests conducted, vaccinations, suspected vs. infected cases, fatalities, active cases, and hospitalizations. However, specific data for the request (number of reported COVID-19 cases, recoveries, and fatalities over time for specific dates) is not directly visible on the current page. For a thorough analysis and comparison with the SIR model for 2024, it would be necessary to have detailed historical data and future projections, which are not available on this website [24]. The information provided on the site is valuable for understanding the current and past situation but does not extend to future predictions like those for 2024. For predictive analysis for the year 2024, it would typically involve extrapolating current trends using mathematical models, considering factors like vaccination rates, natural immunity development, virus mutation rates, and public health policies. However, such predictions would inherently come with a degree degree of uncertainty due to the unpredictable nature of these factors.

5. Conclusion

The SIR model's application to the Albanian COVID-19 data provides valuable insights but also highlights the model's limitations in real-world scenarios. It underscores the need for adaptive strategies in public health planning and the importance of continuously updating models with real-time data. This case study demonstrates the significant role of mathematical modeling in public health, offering a framework for predicting disease spread and informing policy decisions.This article synthesizes the discussion and analysis conducted on the application and limitations of the SIR model in the context of Albania's experience with COVID-19, emphasizing the model's role and potential in public health management. The implementation of the Susceptible-Infected-Recovered (SIR) mathematical model in analyzing the COVID-19 pandemic in Albania provides valuable insights into the dynamics of the disease, as well as the effectiveness of various public health interventions. This study demonstrates the critical role that mathematical modeling plays in public health decision-making, especially in the context of managing and understanding public health crises like the COVID-19 pandemic. Through the integration of real-world data, including vaccination rates and population statistics, the study effectively simulates the progression of the pandemic in Albania [25].

The comparison between the model's predictions and the actual epidemiological data from Albania reveals both the strengths and limitations of using the SIR model in real-world scenarios. It underscores the importance of adaptive strategies in public health planning and the need for continuously updating models with real-time data to improve accuracy and relevance. Furthermore, the study's projection for the year 2024 emphasizes the evolving nature of the pandemic and offers a forward-looking perspective that is essential for long-term planning and preparation in public health policy. It highlights that while mathematical models like the SIR provide a structured framework for understanding and predicting the course of an epidemic, actual data can deviate due to a multitude of variable real-world factors. In conclusion, this case study of Albania's experience with COVID-19 underlines the significance of mathematical models in public health. It provides a comprehensive example of how such models can be effectively used to analyze data, aiding in the development of informed responses to health crises. However, it also cautions against the sole reliance on these models, advocating for their use in conjunction with other epidemiological and public health expertise to ensure comprehensive and effective public health strategies.

References

[1] M. Keeling and L. Danon, "Mathematical modelling of infectious diseases," British Medical Bulletin, 2009. [Online]. Available: https://consensus.app/papers/modelling-diseases-keeling/946b6ec7ef52582a9901085458cedd31/?utm_source=chatgpt.

[2] E. Vergu, "Modeling Infectious Diseases.," Strad Research, 2020. [Online]. Available: https://consensus.app/papers/modeling-diseases-vergu/a7a8ae37cbe85f7c8018c16f916970ff/?utm_source=chatgpt.

[3] E. Dishmema and L. Hanelli, "A SIR Model for Measles Disease Case for Albania.," International Journal of Innovative Research in Engineering & Management., 2019. [Online]. Available: https://consensus.app/papers/model-measles-disease-case-albania-dishmema/343d9a1ca0c3579ea8356b10fcc2b4a9/?utm_source=chatgpt.

[4] W. Kermack and A. McKendrick, "A Contribution to the Mathematical Theory of Epidemics," *Proceedings of the Royal Society A,* vol. 115, no. 772, pp. 700-721, 1927.

[5] H. W. Hethcote, "The Mathematics of Infectious Diseases.," *SIAM Review,* vol. 42, no. 4, pp. 599-653., 2000.

[6] O. Diekmann, H. Heesterbeek and T. Britton, "Mathematical Tools for Understanding Infectious Disease Dynamics," *Princeton University Press,* 2013.

[7] M. Y. Li, "An Introduction to Mathematical Modeling of Infectious Diseases," *Springer,* 2018.

[8] D. J. Daley and J. Gani, "Epidemic Modelling: An Introduction," *Cambridge University Press,* 1999.

[9] H. W. Hethcote, "Qualitative Analyses of Communicable Disease Models," *Mathematical Biosciences,* vol. 28, no. 3-4, pp. 335-356, 1976.

[10] N. T. Bailey, "The Mathematical Theory of Infectious Diseases and its Applications," *Griffin,* 1975.

[11] L. Boden and I. McKendrick, "Model-Based Policymaking: A Framework to Promote Ethical “Good Practice” in Mathematical Modeling for Public Health Policymaking.," Frontiers in Public Health., 2017. [Online]. Available: https://consensus.app/papers/modelbased-policymaking-framework-promote-ethical-good-boden/821e35a8f96250ba9a1479d2822c93db/?utm_source=chatgpt.

[12] F. Brauer, C. Castillo-Chavez and Z. Feng, "Mathematical Models in Epidemiology," *Springer,* 2019.

[13] M. J. Keeling and P. Rohani, "Modeling Infectious Diseases in Humans and Animals," *Princeton University Press,* 2008.

[14] J. D. Murray, "Mathematical Biology: I. An Introduction," *Springer,* 2002.

[15] V. Capasso and G. Serio, "A generalization of the Kermack-McKendrick deterministic epidemic model," *Mathematical Biosciences,* vol. 42, no. 1-2, pp. 43-61, 1978.

[16] P. E. Fine, "Herd Immunity: History, Theory, Practice," *Epidemiologic Reviews,* vol. 15, no. 2, pp. 265-302, 1993.

[17] R. M. Anderson and R. M. May, Infectious Diseases of Humans: Dynamics and Control., Oxford University Press., 1991.

[18] N. C. Grassly and C. Fraser, "Mathematical models of infectious disease transmission," *Nature Reviews Microbiology,* vol. 6, no. 6, pp. 477-487, 2008.

[19] J. A. Heesterbeek and K. Dietz, "The concept of Ro in epidemic theory," *Statistica Neerlandica,* vol. 50, no. 1, pp. 89-110, 1996.

[20] I. M. Longini, E. Ackerman and L. R. Elveback, "An optimization model for influenza A epidemics," *Mathematical Biosciences,* vol. 38, no. 1-2, pp. 141-157, 1978.

[21] R. M. Anderson and R. M. May, "Directly transmitted infectious diseases: Control by vaccination," *Science,* vol. 215, no. 4536, pp. 1053-1060, 1982.

[22] C. Fraser, C. A. Donnelly, S. Cauchemez and a. et, "Pandemic potential of a strain of influenza A (H1N1): Early findings," *Science,* vol. 324, no. 5934, pp. 1557-1561, 2009.

[23] N. M. Ferguson, D. A. Cummings, C. Fraser and a. et, "Strategies for containing an emerging influenza pandemic in Southeast Asia," *Nature,* vol. 437, no. 7056, pp. 209-214, 2005.

[24] World Health Organization, "Coronavirus disease (COVID-19) pandemic," 2020. [Online]. Available: www.who.int/emergencies/diseases/novel-coronavirus-2019.

[25] Centers for Disease Control and Prevention, "COVID-19: Cases, Data, and Surveillance," 2020. [Online]. Available: www.cdc.gov/coronavirus/2019-ncov/cases-updates/index.html.